

13.5 EXERCISES

HOMEWORK KEY

- = **WORKED-OUT SOLUTIONS**
on p. WS1 for Exs. 13, 31, and 45
- ★ = **STANDARDIZED TEST PRACTICE**
Exs. 2, 28, 41, 47, and 48
- ◆ = **MULTIPLE REPRESENTATIONS**
Ex. 45

4 PRACTICE AND APPLY

Assignment Guide

Answer Transparencies
available for all exercises

Basic:

Day 1: SRH p. 980 Exs. 21, 23, 26, 28
pp. 886–888
Exs. 1–6, 12–14, 18–20, 27–32,
43–47, 50, 53, 56, 59, 62, 64

Average:

Day 1: pp. 886–888
Exs. 1, 2, 7–9, 14–16, 21–23, 27, 28,
32–34, 38–41, 43–48, 51, 54, 57, 60,
63, 66

Advanced:

Day 1: pp. 886–888
Exs. 1, 2, 10, 11, 16, 17, 24–28,
32–49*, 52, 55, 58, 61, 65, 67

Block:

pp. 886–888
Exs. 1, 2, 7–9, 14–16, 21–23, 27, 28,
32–34, 38–41, 43–48, 51, 54, 57, 60,
63, 66 (with 13.4)

Differentiated Instruction

See *Algebra 2 Best Practices Toolkit*
for suggestions on addressing the
needs of a diverse classroom.

Homework Check

For a quick check of student under-
standing of key concepts, go over the
following exercises:

Basic: 4, 6, 20, 30, 43, 45

Average: 8, 22, 32, 44, 45

Advanced: 10, 26, 36, 44, 46

Extra Practice

- Student Edition, p. 1022
- Chapter 13 Resource Book:
Practice levels A, B, C, pp. 51–53

Practice Worksheet

An easily-readable reduced
practice page (with answers)
for this lesson can be found
on p. 850C.

SKILL PRACTICE

- A** 1. **VOCABULARY** What information do you need to use the law of sines?
two angle measures and the length of a side or an angle opposite one of two given sides
2. **★ WRITING** Suppose a , b , and A are given for $\triangle ABC$ where $A < 90^\circ$. Under what conditions would you have no triangle? one triangle? two triangles? **See margin.**

EXAMPLES
1, 2, 3, and 4
on pp. 882–884
for Exs. 3–28

2. when the height of the triangle is greater than the side opposite the given angle; when the height is equal to the side opposite the given angle or when the side opposite the given angle is greater than the other given side; when the height of the triangle is less than the side opposite the given angle and this is less than the other given side

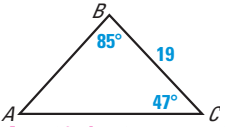
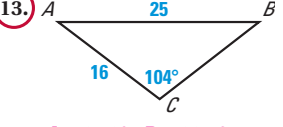
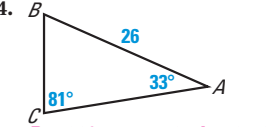
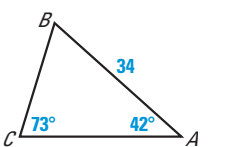
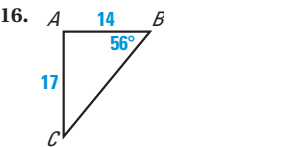
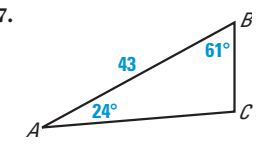
21. $B \approx 54.1^\circ$,
 $C \approx 87.9^\circ$,
 $c \approx 30.8$, or
 $B \approx 125.9^\circ$,
 $C \approx 16.1^\circ$, $c \approx 8.6$

26. $A \approx 116.6^\circ$,
 $C \approx 42.4^\circ$,
 $a \approx 42.4$, or
 $A \approx 21.4^\circ$,
 $C \approx 137.6^\circ$,
 $a \approx 17.3$

IDENTIFYING CASES State the case (AAS, ASA, or SSA) applicable to the given measurements. Then decide whether the measurements determine *one triangle, two triangles, or no triangle*.

- | | | |
|--|---|--|
| 3. $A = 112^\circ$, $a = 9$, $b = 4$
SSA; one triangle | 4. $A = 40^\circ$, $C = 75^\circ$, $c = 20$
AAS; one triangle | 5. $A = 52^\circ$, $a = 32$, $b = 42$
SSA; no triangle |
| 6. $A = 37^\circ$, $a = 8$, $b = 14$
SSA; no triangle | 7. $A = 28^\circ$, $B = 64^\circ$, $c = 55$
ASA; one triangle | 8. $A = 149^\circ$, $a = 7$, $b = 10$
SSA; no triangle |
| 9. $B = 34^\circ$, $b = 5$, $a = 16$
SSA; no triangle | 10. $B = 70^\circ$, $b = 85$, $c = 88$
SSA; two triangles | 11. $C = 48^\circ$, $c = 28$, $b = 20$
SSA; one triangle |

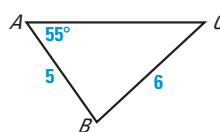
SOLVING TRIANGLES Solve $\triangle ABC$.

- | | | |
|--|---|---|
| 12. 
$A = 48^\circ$, $b \approx 25.5$, $c \approx 18.7$ | 13. 
$A \approx 37.6^\circ$, $B \approx 38.4^\circ$, $a \approx 15.7$ | 14. 
$B = 66^\circ$, $a \approx 14.3$, $b \approx 24.0$ |
| 15. 
$B = 65^\circ$, $a \approx 23.8$, $b \approx 32.2$ | 16. 
$A \approx 80.9^\circ$, $C \approx 43.1^\circ$, $a \approx 20.2$ | 17. 
$C = 95^\circ$, $a \approx 17.6$, $b \approx 37.8$ |

SOLVING TRIANGLES Solve $\triangle ABC$. (Hint: Some of the “triangles” have no solution and some have two solutions.)

- | | | |
|--|--|--|
| 18. $A = 73^\circ$, $a = 18$, $b = 11$
$B \approx 35.8^\circ$, $C \approx 71.2^\circ$, $c \approx 17.8$ | 19. $A = 26^\circ$, $C = 35^\circ$, $b = 13$
$B \approx 119^\circ$, $a \approx 6.5$, $c \approx 8.5$ | 20. $B = 102^\circ$, $C = 43^\circ$, $b = 21$
$A \approx 35^\circ$, $a \approx 12.3$, $c \approx 14.6$ |
| 21. $A = 38^\circ$, $a = 19$, $b = 25$ | 22. $A = 55^\circ$, $B = 64^\circ$, $c = 34$
$C \approx 61^\circ$, $a \approx 31.8$, $b \approx 34.9$ | 23. $A = 114^\circ$, $a = 15$, $b = 10$
$B \approx 37.5^\circ$, $C \approx 28.5^\circ$, $c \approx 7.8$ |
| 24. $C = 98^\circ$, $c = 29$, $a = 33$
no triangle | 25. $A = 49^\circ$, $B = 32^\circ$, $b = 44$
$C \approx 99^\circ$, $a \approx 62.7$, $c \approx 82.0$ | 26. $B = 21^\circ$, $b = 17$, $c = 32$ |

27. **ERROR ANALYSIS** Describe and correct the error in finding the measure of angle C in the triangle below.



The sides were not paired with their opposite angles;
 $\frac{\sin C}{5} = \frac{\sin 55^\circ}{6}$,
 $\sin C = \frac{5 \sin 55^\circ}{6} \approx 0.6826$, $C \approx 43.0^\circ$.

$$\frac{\sin C}{6} = \frac{\sin 55^\circ}{5}$$

$$\sin C = \frac{6 \sin 55^\circ}{5} \approx 0.9830$$

$$C \approx 79.4^\circ$$

28. **★ MULTIPLE CHOICE** What is the side length c in $\triangle ABC$ if $A = 32^\circ$, $C = 67^\circ$, and $b = 31$ ft? **B**

- (A) 16.6 ft (B) 28.9 ft (C) 33.3 ft (D) 57.8 ft

EXAMPLE 5 B
on p. 885
for Exs. 29–41

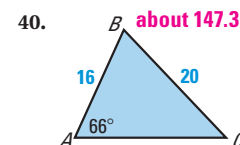
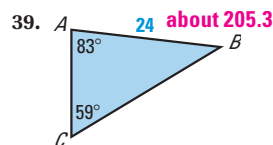
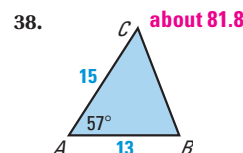
FINDING AREA Find the area of $\triangle ABC$ with the given side lengths and included angle.

29. $B = 124^\circ$, $a = 9$, $c = 11$
about 41.0
32. $C = 79^\circ$, $a = 25$, $b = 17$
about 208.6
35. $A = 130^\circ$, $b = 23$, $c = 20$
about 176.2

30. $A = 68^\circ$, $b = 13$, $c = 7$
about 42.2
33. $B = 57^\circ$, $a = 9$, $c = 5$
about 18.9
36. $B = 60^\circ$, $a = 19$, $c = 14$
about 115.2

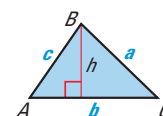
31. $A = 34^\circ$, $b = 29$, $c = 36$
about 291.9
34. $C = 96^\circ$, $a = 7$, $b = 15$
about 52.2
37. $C = 29^\circ$, $a = 38$, $b = 31$
about 285.6

FINDING AREA Find the area of $\triangle ABC$.



41. **★ MULTIPLE CHOICE** What is the area of $\triangle ABC$ if $B = 52^\circ$, $a = 29$, and $c = 24$? **A**
(A) 274 units² (B) 348 units² (C) 548 units² (D) 696 units²

- C** 42. **CHALLENGE** Using the triangle shown at the right as a reference, derive the formulas for the area of a triangle given on page 885. Then use the area formulas to derive the law of sines. **See margin.**



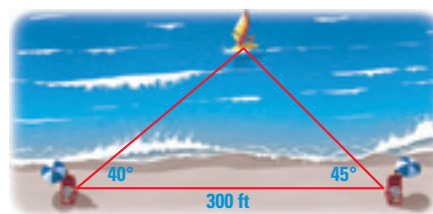
PROBLEM SOLVING

EXAMPLE 1 A
on p. 882
for Ex. 43

43. about 193.6 ft,
about 212.9 ft

43. **LIFEGUARDS** Two lifeguards are watching a windsurfer. Use the information in the diagram to find the distance from each lifeguard to the windsurfer.

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EXAMPLE 2
on p. 883
for Ex. 44

44. **NEW YORK CITY** You are on the observation deck of the Empire State Building looking at the Chrysler Building. When you turn 145° clockwise, you see the Statue of Liberty. You know that the Chrysler Building and the Empire State Building are about 0.6 mile apart and that the Chrysler Building and the Statue of Liberty are about 5.7 miles apart. Estimate the distance between the Empire State Building and the Statue of Liberty. **about 5.2 mi**

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EXAMPLE 5
on p. 885
for Exs. 45–46

45. **◆ MULTIPLE REPRESENTATIONS** You are fertilizing a triangular garden. One side of the garden is 62 feet long and another side is 54 feet long. The angle opposite the 62 foot side is 58° .
- Drawing a Diagram** Draw a diagram to represent this situation. **See margin.**
 - Solving a Triangle** Use the law of sines to solve the triangle you drew in part (a). **third side: about 71.6 ft; other angles: about 47.6° , about 78.4°**
 - Applying a Formula** One bag of fertilizer covers an area of 200 square feet. How many bags of fertilizer will you need to cover the entire garden? **9 bags**

42. *Sample answer:* $\sin A = \frac{h}{c}$, $h = c \sin A$. To compute the area of a triangle use the equation $A = \frac{1}{2}bh$ and by substituting for h you would get $A = \frac{1}{2}bc \sin A$. You get similar results by using angles B and C ($\sin C = \frac{h}{a}$, $h = a \sin C \rightarrow A = \frac{1}{2}ab \sin C$). Since all of these equations represent the area of the same triangle, you can set them equal to each other. This results in $\frac{1}{2}bc \sin A = \frac{1}{2}ac \sin B = \frac{1}{2}ab \sin C$. If you simplify this, you end up with $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$, which is the law of sines.

Avoiding Common Errors

Exercises 3–11 Some students may try to find two different triangles for the AAS and ASA cases. Remind them that only the SSA case does not determine exactly one triangle, which means it is the only case in which there may be 0, 1, or 2 triangles. Use the term “ambiguous case” to emphasize this point.

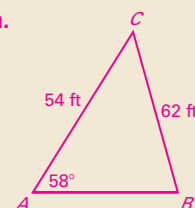
Mathematical Reasoning

Exercises 18–26 Sometimes students have trouble solving triangles because they do not know in what order they should try to find the missing parts. Have them work individually or with partners to develop an organized strategy for solving a triangle for each of the cases discussed in this lesson. Encourage them to find effective ways to present this information. A flow chart is one way that will work well. After you discuss the SAS and SSS cases in Lesson 13.6, students should go back and add that information to what they have put together for the AAS, ASA, and SSA cases here.

Study Strategy

Exercises 29–41 Some students may benefit from a hint for remembering the new formula for the area of a triangle. If they forget whether the formula includes $\sin A$ or $\cos A$, suggest that they try this test: What if the included angle is 90° ? Since $\sin 90^\circ = 1$, if $C = 90^\circ$, this would give $\text{Area} = \frac{1}{2}ab$. This says that the area of a right triangle is half the product of the lengths of its legs, which is correct. But since $\cos 90^\circ = 0$, using $\cos C$ in the formula would give an area of 0, which cannot be correct.

45a.



5 ASSESS AND RETEACH

Daily Homework Quiz

Transparency Available

Solve $\triangle ABC$ with the given parts.

1. $A = 112^\circ$, $a = 24$, $B = 29^\circ$

$C = 39^\circ$, $b \approx 12.5$, $c \approx 16.3$

2. $A = 96^\circ$, $a = 16$, $b = 7$

$B \approx 25.8^\circ$, $C \approx 58.2^\circ$, $c \approx 13.7$

Decide if the given measurements determine *one triangle*, *two triangles*, or *no triangle*.

3. $A = 71^\circ$, $a = 45$, $c = 47$

two triangles

4. $C = 85^\circ$, $b = 33$, $c = 28$

no triangle

5. Two sides of a sports pennant are each 18 inches long, and the angle between these two sides is 50° . What is the area of the pennant? **about 124 in.²**

Online Quiz

Available at classzone.com

Diagnosis/Remediation

- Practice A, B, C in Chapter 13 Resource Book, pp. 51–53
- Study Guide in Chapter 13 Resource Book, pp. 54–55
- Practice Workbook, pp. 189–190
- @HomeTutor

Challenge

Additional challenge is available in the Chapter 13 Resource Book, p. 58.

47b. See Additional Answers beginning on p. AA1.

48a. Sample answer:
 $A = 24 \sin x$, where the length of the sides of the triangle are 6 and 8.

48b. No; x increases to 90° where the equation reaches a maximum and then begins to decrease.

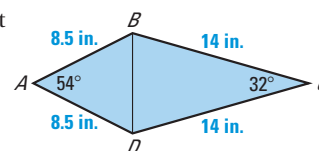
C

46. MULTI-STEP PROBLEM Quadrilateral $ABCD$ shown at the right is a kite.

a. Find the area of $\triangle ABD$. **about 29.2 in.²**

b. Find the area of $\triangle BCD$. **about 51.9 in.²**

c. What is the area of the kite? **about 81.1 in.²**



B **47. ★ SHORT RESPONSE** A building is constructed on top of a cliff that is 300 meters high. A person standing on level ground below the cliff observes that the angle of elevation to the top of the building is 72° , and the angle of elevation to the top of the cliff is 63° .

- a. How far away is the person from the base of the cliff? **about 152.9 m**
- b. Describe two different methods you can use to find the height of the building. Use one of these methods to find the building's height. **See margin.**

48. ★ EXTENDED RESPONSE Use a graphing calculator to explore how the included angle in the formulas on page 885 affects a triangle's area.

- a. **Model** Choose lengths for two sides of the triangle. Let x represent the measure (in degrees) of the included angle. Write an equation that gives the triangle's area y as a function of x .
- b. **Graphing Calculator** Enter the equation from part (a) into a graphing calculator. Use the *table* feature to examine values of the area for $0^\circ < x^\circ < 180^\circ$. Does the area always increase as x increases? **Explain.**
- c. **Interpret** What value of x maximizes the triangle's area? What is the maximum area, and how is it related to the side lengths you chose in part (a)? **90° . Sample answer: 24, it is equal to one-half of the product of the side lengths.**

49. CHALLENGE The distance between Mercury and the sun is approximately 36 million miles. The distance between Earth and the sun is approximately 93 million miles. If on a certain day the angle (measured from Earth) between the sun and Mercury is 22° , what are the possible distances between Mercury and Earth? **about 95 million mi or about 77 million mi**

MIXED REVIEW

Perform the indicated operation. (p. 420)

50. $6\sqrt{13} - \sqrt{13}$ **$5\sqrt{13}$**

51. $5(250)^{1/3} - 10(54)^{1/3} - 5(2)^{1/3}$ **$8\sqrt[3]{10}$**

53. $5(20)^{1/2} - 3(45)^{1/2}$ **$5\sqrt{2}$**

54. $9\sqrt[3]{56} - \sqrt[3]{189}$ **$15\sqrt[3]{7}$**

55. $-6(88)^{1/3} + 9(297)^{1/3}$ **$15(11)^{1/3}$**

Perform the indicated operation and simplify. (p. 573)

56. $\frac{5x^5}{x^3y} \cdot \frac{xy}{20xy^2}$ **$\frac{x^2}{4y^2}$**

57. $\frac{40x^3y^3}{2xyz} \div \frac{10xy}{x^2yz^3}$ **$2x^3y^2z^2$**

58. $\frac{3x^2 - 9}{x - 1} \cdot \frac{x + 7}{6x^2 - 18}$ **$\frac{x + 7}{2(x - 1)}$**

59. $\frac{4x^2 - 16}{x^2 - 25} \cdot \frac{x + 5}{4x - 8}$ **$\frac{x + 2}{x - 5}$**

60. $\frac{x^2 + 5x + 6}{3x^2 + 13x + 14} \div \frac{(x + 3)}{3x + 7}$

61. $\frac{6x^2 + 11x + 4}{2x^2 + 3x - 1} \div \frac{6x - 8}{x + 1}$
 $\frac{(3x + 4)(2x + 1)(x + 1)}{2(2x^2 + 3x - 1)(3x - 4)}$

Solve the equation for θ . (p. 875)

62. $\cos \theta = 0.75$; $270^\circ < \theta < 360^\circ$ **about 318.6°**

63. $\cos \theta = -0.6$; $180^\circ < \theta < 270^\circ$ **about 233.1°**

64. $\cos \theta = -0.35$; $90^\circ < \theta < 180^\circ$ **about 110.5°**

65. $\cos \theta = 0.92$; $270^\circ < \theta < 360^\circ$ **about 336.9°**

66. $\cos \theta = -0.28$; $180^\circ < \theta < 270^\circ$ **about 253.7°**

67. $\cos \theta = 0.47$; $270^\circ < \theta < 360^\circ$ **about 298.0°**

PREVIEW

Prepare for Lesson 13.6 in Exs. 62–67.

